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Responsibilities of a Teacher in a Harmonic Cycle of Problem Solving and Problem Posing

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Early math experiences can be challenging for young children. Although it seems that planning a lesson on solving and posing a problem for a young child is a daunting task, it is actually simple if a harmonic cycle is exercised. This harmonic cycle, in fact, embeds within in it a variety of teacher's responsibilities that can expand our understanding of what a teacher needs to attend to when it comes to planning, teaching, and evaluating a child's progress. This article will present responsibilities that a teacher has in a harmonic cycle, which is reflected through an interaction between a teacher and a child on problem posing and problem solving.

KEY WORDS: problem solving and posing; teacher's responsibilities; early childhood; mathematics.

INTRODUCTION

One of my students once wrote: "I have found that sometimes it can be difficult to get children excited and eager to learn about Math ..." If early math experiences are difficult to have, how about problem posing and problem solving? Could a 7-year-old child in first or second grade know how to solve and then to pose a problem?

It may seem as though the process of finding a solution to a problem is shown backwards in the previous sentence and in the title. Surely, problem posing precedes problem solving. Please note, however, that this article demonstrates the responsibilities of a teacher in helping a young child to learn *first* how to solve a problem that a teacher develops. With the understanding of the process of problem solving with concrete materials, the child *then* is encouraged to pose a problem on his own. This explains the logic used when choosing the title for this paper "Problem Solving and Problem Posing" and for maintaining this same order throughout the paper.

Could the processes of problem solving and problem posing be taught to young children in a developmentally appropriate fashion? Although it seems that planning a lesson on solving and posing a problem for a young child is a daunting task, it is actually simple if a harmonic cycle is exercised. This harmonic cycle, in fact, embeds within in it a variety of teacher's roles that can expand our understanding of the responsibility of planning, teaching, and evaluating a child's progress.

This article, comprised of three parts, will present that a teacher plays in a harmonic cycle reflected through an interaction between a teacher and a child on problem solving and problem posing: The first part presents the dialogues between John (the teacher) and Tyler (the child), which is followed by an analysis of the interaction. The third and final part summarizes the teacher's responsibilities in facilitating the child's inquiry.

Part One

John's intimate knowledge of Tyler made him aware that Tyler knew simple addition and subtraction, but had not yet been exposed to problem

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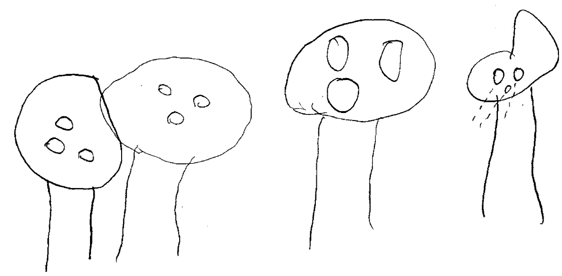
solving and problem posing. The conversation below was videotaped and then transcribed by John.

John: "Tyler, would you help me solve a problem?"
Tyler: "Sure!"
John: "Okay. There's a farmer who has four apple trees in his orchard. Each of the apple trees has three ripe apples on it. A boy comes along and eats three ripe apples. Now, how many ripe apples are there in the whole orchard?"
Tyler: "Hmm." He scratches his head.
John: "I have some apples and pieces of green construction paper. We could pretend that each piece of green construction paper is a tree. Would you like to try to use these apples and green paper to get an answer to this problem?"
Tyler: "OK."
John: "I am placing three apples on this piece of construction paper. What should we do next?"
Tyler: "We could put three apples on each of these trees." He pointed to the remaining three pieces of the green construction paper.
John: "That's right. Can you do that for me?" Tyler placed three apples on each of the remaining three pieces of construction paper. "Could you please let me know how many apples these four trees have altogether?" The child started counting all the apples on the green paper "trees."
Tyler: "Twelve."
John: "Yes. Now, how many ripe apples are there if the boy eats three of them?"
Tyler: He moved one "apple tree" to his left side and then counted the rest of the apples remaining on the three trees. "Nine."
John: "Now, let's see whether 9 is a correct by drawing a picture. Do you want to try it with me?"
Tyler: "Okay."
John: "Do you remember how many apple trees the farmer had in his orchard?"
Tyler: "Four."
John: "Yes. We can draw four apple trees on the paper. Does that sound like a good idea?"
Tyler: "Yeah."
John: "I'll draw two trees, and you can draw two trees." They worked together to draw the four trees. "Now, how many ripe apples were on each tree? Do you remember?"
Tyler: "Three."
John: "Good. Shall we add three apples to each of our trees?"
Tyler: "OK."
John: "Let's see how many apples these four trees have altogether."
Tyler: "Twelve!"
John: "Now, here comes a boy who eats three apples from one tree. How would you show that the apples have been consumed?"
Tyler: "Use this." He drew a few dotted lines, whose direction was downward to the "ground."
John: "OK. How many apples are there now?"
Tyler: "Nine." (see Picture 1)
John: "Thanks."



Picture 1.

John let Tyler take a bathroom break. After he returned for the second session, John provided Tyler with a problem that involved only subtraction.

John: "Do you want to try to solve a different problem now?"
Tyler: "Sure."
John: "A farmer has twenty apples in his basket. First, he gave two apples to Tom. Later, he gave seven apples to Sally. On the way home, the farmer ate three apples. How many apples are left in the basket? Could you show me how to solve the problem?"

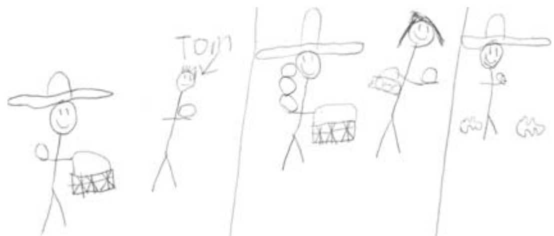
Tyler counted to make sure there were twenty apples on the table, and he grouped them all together. He moved one piece of construction paper to the side and placed two apples on it. Next, he set a second piece of construction paper beside the first and placed seven apples on it.

Tyler: "How many did the farmer eat?"
John: "He ate three apples."

Tyler set a third piece of construction paper beside the other two, and he place three apples on it. He counted the apples that remained in the original pile on the table.

Tyler: "There are eight left in the basket."
John: "That's right."
Tyler: Tyler counted the apples on the three pieces of construction paper. "Twelve plus eight equals twenty."
John: "Very good. Can you explain to me how you used the construction paper?"
Tyler: "This one is Tom, this one is Sally, and this one is the farmer. Tom got two apples, Sally got seven apples, and the farmer ate three."
John: "Thanks. Could you draw pictures to check the answer?"

Tyler worked for approximately 5 minutes on his drawing. His drawing depicted all three scenarios (two apples to Tom, seven apples to Sally, and three apples to the farmer) on one piece of paper. Each scenario was separated from the others with a line. John realized that Tyler had not drawn anything to



Picture 2.

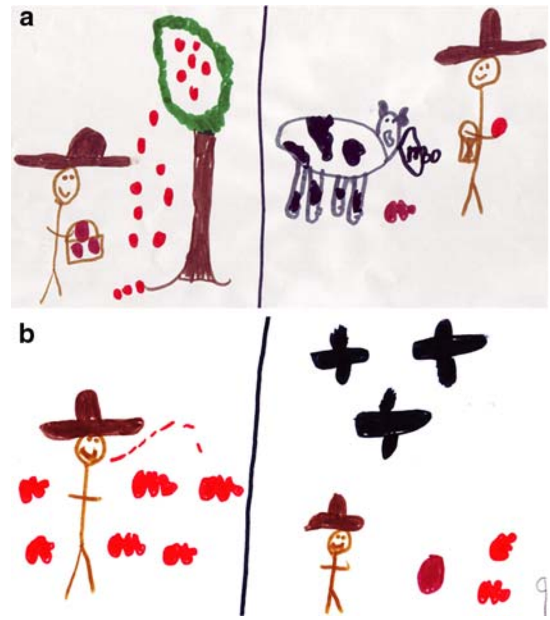
represent the initial basket full of twenty apples (see Picture 2).

John: "How would these drawings help you solve the problem?"
Tyler: "Because these are all the apples he gave away."
John: "But how many did we start out with?"
Tyler: "Twenty."
John: "That's right. Do you think this picture would help us solve the problem if we knew that the basket had twenty apples in it to begin with?"
Tyler: "Yeah."
John: "I do, too. There isn't really room to put another picture on this piece of paper. Do you want to draw that basket with twenty apples in it on another piece of paper?"
Tyler: "Nah."
John: "Okay, you don't have to. You understand why it would work better to draw all the apples first, right?"
Tyler: "Yea. So we know how many to subtract from."
John: "Okay. Good."

After another break, John brought Tyler back to the table for another session. The following are the details from this third session:

John: "Tyler, do you think you can make up a story problem of your own now?"
Tyler: "Write it or draw it?"
John: "You can draw it first. Then, you tell me what it is and I'll write it. Then you are going to solve it by yourself.
Tyler: "Okay." He thinks about the possibilities for about twenty seconds. He points to the picture he created a few moments ago. "Okay. I'm going to do a back-track on this one. Like, a farmer has twenty apples, and the cows ate two of them. Then the farmer ate six. And the crows came down and took three.
John: "Is that it?"
Tyler: "Yeah."
John: "Okay. What do you want to find out, how many apples the farmer has left?"
Tyler: "Yeah, how many he has left."
John: "Alright."

Tyler worked on his drawing for about 10 min. He remembered to include the total amount of apples that the farmer started with in his first



Picture 3.

picture. He then drew each of the three scenarios (the cow eating two apples (see Picture 3a), the farmer eating six apples and the crows getting three apples, see Picture 3b) in partitioned pictures within the same group of drawings. When he had finished the drawings, he counted the apples in the two pictures. He subtracted that number from the original twenty apples to find the solution to the problem. He wrote the answer, 9, at the bottom of the second page (see Picture 3b). Then, John encouraged him to use the apples and construction paper to verify the answer.

AN ANALYSIS OF THE RESPONSIBILITIES OF A TEACHER

Charlesworth and Lind (2007) suggested that teachers attend to the following as children engage in problem-solving activities:

a. *Assess*—Where is the child now?
b. *Choose objective*—What should she learn next?
c. *Plan experiences*—What should the child do to accomplish these objectives?
d. *Select materials*—Which materials should be used to carry through the plan?
e. *Teach*—Do the plan and the materials fit the expectation as indicated by the objective(s)?
f. *Evaluate*—Has the child learned what was taught (reached objectives)?

These six steps formulate a harmonic cycle from pre-assessment (knowing the learner) to evaluation (understanding of how the learner has achieved the objectives). According to Reggio Emilia educators, the cycle moves upward spirally as learning becomes more and more complex and intricate (Edwards, Gandini, & Forman, 1998). Planning, selecting materials, and teaching are also central to the achievement of goals/objectives. Although a harmonic learning cycle is core to successful learning, the responsibilities the teacher assumes are essential to making it happen. The following presents an analysis of the vignette provided above in light of the teacher's responsibilities in problem solving and problem solving posing:

Assess. In many instances, assessment, taking place at this stage, is viewed as pre-assessment. The purpose of pre-assessment is to allow the teacher to surmise the extent of background knowledge and level of understanding the child already possesses. A teacher conducts pre-assessment in various forms. Pre-assessment can be obtained through paper–pencil solicitations via printed questions and answers. Pre-assessment information can also be sought in the form of an oral interview or a conversation between a teacher and child. Sometimes, pre-assessment happens prior to designing the curriculum and, sometimes, just before a lesson begins. At any rate, pre-assessment is an integral part of a well-designed curriculum.

From the dialogue provided above, it is clear that John had known Tyler since the child was born. Through frequent interaction with the child, John learned that Tyler had not been exposed to problem posing and problem solving. With respect to Tyler's background knowledge, for instance, John was confident that the child was interested in apples because he had worked with Tyler a month ago on a project related to a science concept—the growth cycle of an apple. Based on all these bits of information, John designed the problems that he believed would fit the child's level of understanding and allow a review of addition and subtraction concepts.

Choose objective. "Problem solving should not be an isolated part of the curriculum but should involve all Content Standards" (National Council of Teachers of Mathematics, 2004). In contemporary society, teaching and learning is implemented in compliance with standards, set by the state or school district. In every academic year, a school is held accountable for students' achievement. Gauging the level of achievement is discerned by means of standardized tests. For example, in the state of Indiana, an ISTEP (Indiana Statewide Testing for Educational Progress) test is administered every spring for children, grades 3–10. With regards to problem posing and problem solving, Indiana's Academic Standards (Indiana Department of Education, 2005) call for students to be able to pose a problem, use various means to solve a problem, reason its solution, and verify the answer. For example: Pertaining to the first grade, since Tyler is a first grader, the following standards are expected to be achieved by the end of the academic year (see Table I).

The mandate of satisfying given standards requires the teacher to be mindful in designing the curriculum. If focusing too much on standards, a lesson could become an overwhelming task for young children to complete; the lesson is inflated by an attempt to match many standards. However, if only a few standards are attended to, the accountability may be questioned.

The following will present the reader with an idea of how the interaction between John and Tyler satisfied those standards (see Table II).

Plan experiences. The source of power that drives children toward problem solving and problem posing is their curiosity. Emerging curiosity helps sustain children's persistence as they want to find out solutions to problems that they generate and that interest them. Children's persistent investigation requires an environment in which their curiosity and determination to find solutions may possibly grow and develop. Such an environment should also concurrently

Table I. Relevant Indiana's Academic Standards for First Grade in Problem Posing and Problem Solving

Indiana' Academic Standard	Content
Standard 3: Algebra and functions: 1.3.2	Create word problems that match given number sentences involving addition and subtraction.
Standard 6: Problem solving: 1.6.1	Choose the approach, materials, and strategies to use in solving problems.
Standard 6: Problem solving: 1.6.2	Use tools such as objects or drawings to model problems.
Standard 6: Problem solving: 1.6.3	Explain the reasoning used with concrete objects and pictures.
Standard 6: Problem solving: 1.6.4	Make precise calculations and check the validity of the results in the context of the problem.

Table II. Action in Compliance with the Relevant Indiana's State Academic Standards

Indiana's Academic Standard	Meeting Standards
Standard 3: Algebra and Functions: 1.3.2 Create word problems that match given number sentences involving addition and subtraction.	John developed a word problem involving both addition and subtraction (Session 1) and another addressing only subtraction in (Session 2).
Standard 6: Problem Solving: 1.6.1 Choose the approach, materials, and strategies to use in solving problems.	John offered various materials for Tyler to choose in this learning process, including pencils, color markers, two types of paper, and apples. In addition, John facilitated Tyler's efforts to solve the problem with concrete materials and a drawing.
Standard 6: Problem Solving: 1.6.2 Use tools such as objects or drawings to model problems.	John modeled for the child by using the construction paper as apple trees: "We could pretend that each piece of green construction paper is a tree." He worked with the child to show the understanding of the problem through a drawing: "I'll draw two trees, and you can draw two trees." They worked together to draw the four trees (see Picture 1).
Standard 6: Problem Solving: 1.6.3 Explain the reasoning used with concrete objects and pictures.	In a later interaction with John, John gave Tyler an opportunity to explain the reasoning he had already employed: "Can you explain to me why he used the construction paper?"
Standard 6: Problem Solving: 1.6.4 Make precise calculations and check the validity of the results in the context of the problem.	John employed appropriate strategies to help the child arrive at the answers with the use of drawings (see Pictures 2–4) and guided the child to corroborate the answers by using concrete objects (apples and papers).

facilitate children's knowledge and skills in problem posing and problem solving (Brown et al., 1993; Charlesworth & Lind, 2007; Lowrie, 2002).

It is the teacher who is able to set up an intentional learning environment (Brown et al., 1993). The teacher designs the environment on the basis of children's life experiences and general learning levels that he or she has obtained earlier (Charlesworth & Lind 2007; Lowrie, 2002). If problems or situations are closely connected with what they know and experience, children's levels of confidence and motivation tend to be boosted.

The dialogue provided above demonstrates that John had a clear understanding about this connection. He knew that enduring learning is inseparable from children's interest. He connected to what he knew of the child's interest in apples and the child's apple related experiences (There were apples in Tyler's house and he had visited a farm on a fall field trip). What was more was that John provided real apples and fantasy trees (green construction paper). These concrete materials are fun to work with, which might increase the child's motivation in learning. Consequently, Tyler was likely to participate in these problem-solving and problem-posing sessions. Taking all these into consideration, John set up an intentional problem-solving and problem-posing environment focused on apples. The questions that John generated were based on Tyler's daily life experience and his interest.

Select materials. As in John's case, sometimes we know that materials available in a classroom could easily be adopted for the purpose of provocative learning (Buschman, 2003). With an intimate knowledge of the child, John selected real apples, paper, pencils, and crayons. First grade children are still in the pre-operational cognitive development stage. Teachers are, therefore, encouraged to use a wide variety of concrete experiences to help the child learn, including working with manipulatives, working in groups, and going on a field trip (Huitt & Hummel, 2003). Commonly, learning starts with young children's manipulation of concrete materials. In John's case, 20 apples and construction paper were presented as concrete materials. Tyler was able to solve the problems generated by John as well as by himself with the use of the materials. To allow learning to leap to an abstract level, children usually go through a middle point: semi-abstract materials. The semi-abstract forms in John's case were drawings. The concrete materials as well as drawings are representative of symbols that children usually resort to for problem solving and problem posing.

Teach. Teaching and learning took place in three different sessions: In the first session, the teacher developed and posed a problem to the child. The child was helped with various means by the teacher to analyze the problem in order to know the structure of a problem and to solve it. The second session still started with a problem generated by the teacher for further construction of knowledge with the supervision of the teacher to provide assistance, if needed. In the third session, the child applied what he had learned by posing a problem. These efforts formed an intentional problem-solving environment (Brown et al., 1993).

There were different forms of opportunities for Tyler to engage in the new inquiry. The child utilized a mixture of materials as well as a range of interactions with John to develop an understanding of how to solve and pose problems. The use of manipulatives to assist students in solving word problems is fully supported by Skinner (1990). John posed the first question involving addition and subtraction before leading the child to analyze it segment by segment with the use of concrete materials. This analysis not only allowed the child to know about useful strategies applied to solving problems, but also offered an opportunity for the child to reorganize the given problem in a way that he easily understood it (Lowrie, 2002). While the problem-solving exercise was underway, Tyler had an opportunity to have another look at addition and subtraction, knowledge about which he had ideas. Not only were appropriate problems developed that fit the child's needs and understanding level, but also strategies associated with problem solving and problem posing were presented to the child as models. Opportunities were also provided for the child to check the answers to each problem. The National Council of Teachers of Mathematics—Curriculum and Evaluation Standards for School Mathematics (2000) stated, "Ideally, children ... should learn several ways of representing problems and strategies for solving them" (p. 23). Buschman (2003) noticed in his work with children for over 10 years of experience that as children were provided with opportunity to solve problems in their own way and with materials that they would like to choose, they developed their own strategies, which went beyond what adults originally had attempted to do.

Evaluate. Through the three sessions, John closely monitored the learning status of the child, which allowed him to make an informed decision about a next learning step. Planning and teaching without checking to see whether objectives have been achieved leaves the learning cycle incomplete. The teacher checks as teaching goes on. For example, the fact that John asked the child to verify the answers to those problems that he posed as well as the one that the child posed helped the child establish an appropriate habit of mind that one needs to be responsible for what one has done. It also provided an opportunity for the teacher to see the progress that the child had made. John felt confident that Tyler had mastered the concepts of problem solving and problem posing via the drawings that he produced. While supervising Tyler, John detected an area that was worth Tyler's attention:

John: "How would these drawings help you solve the problem?"
Tyler: "Because these are all the apples he gave away."
John: "But how many did we start out with?"
Tyler: "Twenty."
John: "That's right. Do you think this picture would help us solve the problem if we knew that the basket had twenty apples in it to begin with?"
Tyler: "Yeah."
John: "I do, too. There isn't really room to put another picture on this piece of paper. Do you want to draw that basket with twenty apples in it on another piece of paper?"
Tyler: "Nah."
John: "Okay, you don't have to. You understand why it would work better to draw all the apples first, right?"
Tyler: "Yea. So we know how many to subtract from."
John: "Okay. Good."

Later, Tyler remembered to include the total amount of apples that the farmer started with in his first picture (see Pictures 3 and 4). The review of the third drawings informed John that Tyler had developed an idea of how to pose a problem and how to solve a problem appropriately.

CONCLUSION

Tyler did not have an idea about what a word problem was and how to approach it. Owing to various responsibilities that John played in the learning process, Tyler was able to solve and then pose word problems toward the end of three sessions with John. John modeled problem solving and problem posing. The child not only confidently solved problems that John developed, but also was able to generate a problem himself and to rationalize a solution to the problem. The interaction appeared to empower the child to be persistent and learn from mistakes made in the learning process. In this harmonic teaching and learning cycle, John, as a teacher, took a variety of responsibilities before, during, and after teaching about how to solve and pose a problem, including those of an assessor, a planner, a teacher, a facilitator, a guide, a thinker, a supervisor, and an evaluator.

In general, a teacher must take time to get to know his or her students, understand the content of state standards, devise learning objectives to meet stage standards, utilize available materials (Skinner, 1990) and graphic representations (Charlesworth & Lind, 2007), interact appropriately with the child, and evaluate the child's performance. The National Council of Teachers of Mathematics (2000) and the Indiana Department of Education (2005) have called for an increased emphasis on problem-posing and problem-solving activities in mathematics instruction so that students may be able to:

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

Solving problems is not only a goal of learning mathematics, but also an integral part of all mathematics learning (NCTM, 2004). The teacher can ease a child's acquisition of problem-solving and problem-posing skills by understanding the responsibilities that he or she takes for guiding the child's mathematical learning process.

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